The Tradeoff Between the Number of Deployed *p*-Cycles and the Survivability to Dual Fiber Duct Failures

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***Abstract*— *p*-Cycles can attain high capacity efficiency and fast protection switching times in WDM networks. The number of deployed *p*-cycles and the ability to survive dual fiber duct failures are further important characteristics which are considered in a pan-European network case study. We show that the dual failure restorability and the protection capacity can vary significantly for cycle-configurations with different numbers of deployed *p*-cycles.**

I. Introduction

The *p*-cycle protection concept is a promising approach for survivable WDM networks, since high capacity efficiency and fast protection switching times can be achieved [1,2]. WDM *p*-cycles guarantee survival of any single fiber duct failure (e.g., caused by a backhoe). But for larger networks, dual failures—though much less probable than single failures—should be taken into account [3,4]. Therefore the behavior of *p*-cycles in presence of dual fiber duct failures comes to the forefront. Additionally, the configuration of *p*-cycles for a network can optimize the performance in case of double failures.

Fig. 1 (a) depicts a network with one (link) *p*-cycle (a detailed description of the *p*-cycle concept can be found in [1]). This *p*-cycle is able to protect on-cycle links as shown in Fig. 1 (b). Furthermore, a *p*-cycle is able to protect straddling links. A straddling link is an off-cycle link having *p*-cycle nodes as endpoints. In case of a straddling link failure, each *p*-cycle can protect two working paths on the link by providing the two alternative paths around the *p*-cycle as shown in Fig. 1 (c). For both on-cycle and straddling links the protection switching can be made very fast, since only the nodes neighboring the failure need to perform real-time actions.

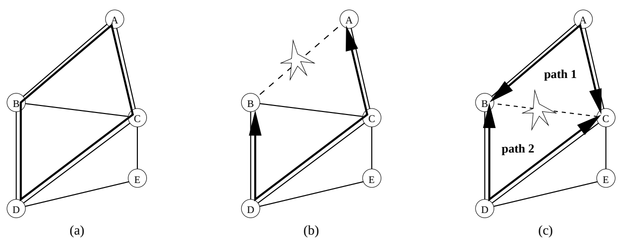

Fig. 1. A network with one link *p*-cycle (a) which can protect on-cycle links (b) and straddling links (c).

In a larger network multiple *p*-cycles can be deployed [1,2]. Then, for the general problem, we seek in a two-connected and capacitated network an optimal routing for the demands, and an optimal configuration of *p*-cycles protecting these. Although the problem can be solved as a whole [5], we pursue a two-step approach where first the demand connections are routed through the network and then the *p*-cycles are formed. By this we can use an arbitrary routing algorithm, however, we may attain sub-optimal solutions only.

After the routing procedure, (working) capacity is reserved on the ducts. The spare capacity of the ducts is the remaining available capacity, in which the *p*-cycles are formed. The set of *p*-cycles is chosen such that, for every duct the working connections are protected by *p*-cycles of corresponding capacity.

As a result the network is guaranteed to be protected against single fiber duct failures. Subsequent failures in presence of a single failure can be survived by finding *p*-cycles in the remaining topology (without the failed element). For this the *p*-cycles have to be reconfigured based on the new topology and the new paths (which have been restored). This approach, which is non-disruptive for the working paths, is described in [6].

In this paper we assume that after a first failure the *p*-cycles remain as initially configured. This is applicable in the following cases:

- Reconfigurations are not desired (e.g., adaptive changes of the protection configuration should be avoided).
- A reconfiguration is not possible (e.g., in a fixed *p*-cycle infrastructure [1] or due to lack of capacity).
- The reconfiguration after a first failure is not completed.

In this context we investigate how *p*-cycles perform upon dual fiber duct failures, and how we can improve the performance by modifying the selection of *p*-cycles.

Working capacity protected by a high number of deployed *p*-cycles can be very resilient to double failures, however, it also introduces higher administrative effort for the network operator. Therefore we consider in this paper the tradeoff between this number and the survivability of dual fiber duct failures.

II. *p*-Cycles and Dual Failures

In this section we analyze which dual failures a *p*-cycle network cannot survive. Double failures are treated as ordered events (failure at time t_1, failure at time t_2). The individual failures occur independently, such that the recovery of the first failure at t_1 is completed before t_2.

Multiple failures are guaranteed to survive in networks protected by multiple *p*-cycles, if at most one failure occurs in any individual *p*-cycle. As effectively each cycle "sees" just a single failure, however, for a double failure analysis, we need only to consider occurrences of double failures within a cycle.

In Fig. 2 we analyze the dual failure cases within a single *p*-cycle which can cause the loss of protected working units. We categorize the failures by (F_1 w_1, F_2 w_2), where $F_1, F_2 \in$ {on-cycle, straddling} determine the link type of the two failed links 1 and 2. For a link i the working capacity protected by the *p*-cycle is denoted by w_i, where $w_i \in \{0,1\}$ and $w_i \in \{1,2\}$ for F_i =on-cycle and F_i =straddling, respectively. We assume that after the first failure the *p*-cycle remains potentially accessible for a second failure, e.g., there is no protection switching for a failed on-cycle link with $w_1 = 0$ (a link with no protected working traffic).

Fig. 2 (a) shows a *p*-cycle protecting all the links of the network (five on-cycle and two straddling links). Double failures may not be (fully) survived in the following cases:

1) *Two on-cycle link failures* cause a loss, since the protection path is not available anymore. Fig. 2 (b) depicts this situation. After the failure between nodes C and D is recovered, a second failure on link A-E disrupts both the working traffic on A-E and the protection path for C-D. Note that if link C-D does not carry traffic protected by the *p*-cycle, a switching action is not necessary, but still the traffic on link A-E cannot be protected by this cycle anymore.
2) *An on-cycle failure preceeding a straddling failure* cannot be fully survived (Fig. 2 (c)), since, as in the previous case, the protection path is unaccessible. However, if the *p*-cycle protects only one unit of link B-E and no traffic on link C-D, the traffic on link B-E can survive if it is routed over B-A-E, otherwise it will not.
3) We obtain different situations for *a straddling link failure preceeding an on-cycle link failure* as depicted in Fig. 2 (d)-(f). In any case we experience loss except if the protection path for a single unit on a straddling link is disjoint from the secondly failed link which does not carry working traffic (Fig. 2 (f)).
   The losses occurring as in Fig. 2 (e) can be decremented by one, if the protection path B-A-E is chosen after the first failure. This, however, needs further failure signaling and is not considered.
4) For *double straddling link failures*, loss is also caused as shown in Fig. 2 (g)-(h). If the first failed straddling link carries only one working unit, and the protection path of one working unit of the second path are disjoint, a double failure for this (second) working unit will be survived (Fig. 2 (h)).
   As depicted for the *p*-cycle in Fig. 3 (a) (six on-cycle and three straddling links) we cannot easily guarantee this disjointness. A double failure (B-F,B-E) can be survived as shown in Fig. 3 (b). If the protection switching for B-F at nodes B and F is pre-configured towards C and D, respectively, in the failure case (B-F,C-F) the resulting path B-C-D-F causes the protection for C-F to block, see Fig. 3 (c). If the protection path for B-F is B-A-E-F, this blocking is avoided, however, it is introduced in turn for failure (B-F,B-E). Other network examples can be constructed where a disjointness can never be achieved, e.g. for the cycle with four on-cycle and two straddling links in Fig. 4 (a). Here a double straddling link failure always causes a loss of the secondly failed straddling link (Fig. 4 (b)).

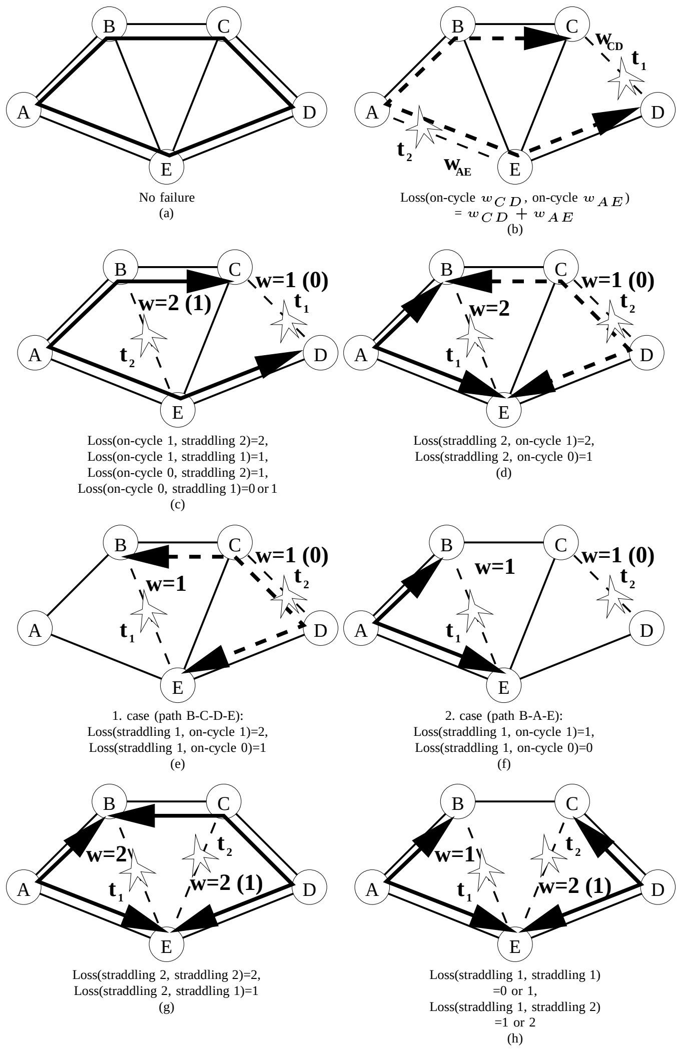

Fig. 2. Double failure cases within a single *p*-cycle

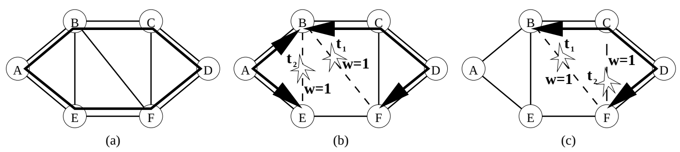

Fig. 3. Dependence of the loss on the protection path

We observe that the loss can be dependent on the sequence of failures, e.g. Loss(straddling 1, straddling 2) can differ from Loss(straddling 2, straddling 1) in Fig. 2 (g)-(h).

The protection switching analyzed so far is based on link information. If nodes also take demand information into account, a better double failure survivability can be achieved. Fig. 4 (c) depicts the situation where a demand protected by a *p*-cycle as in Fig. 2 (a) could survive a double failure, if after the second failure node B switches to the active *p*-cycle. For this, node B requires the knowledge about the association of the demands to the *p*-cycles (and not just the demand-to-link relationships) and a detection that an active *p*-cycle protects a secondly failed link with the same demand. In this paper we do not consider this kind of recovery.

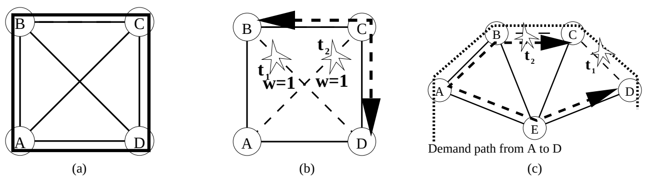

Fig. 4. Particular examples where a double failure cannot be survived

III. *p*-CYCLE SELECTION

This section describes a method to include the number of selected nodes in the mathematical formulation for the optimal combination of *p*-cycles.

We consider WDM networks with full wavelength conversion. WDM nodes are interconnected by fibers (or fiber transmission systems). The wavelength in a fiber that is reserved by a connection or *p*-cycle is called link. A duct (or span) comprises all the fibers between a given pair of nodes.

Recall that we used a bidirectional model in the previous section. As in [2], we now treat ducts as undirected, and fibers, connections, *p*-cycles, and links as unidirectional, where we obtain analogous results as in the previous section.

For the model, each duct in the network is represented by a pair of counterdirectional edges. The network is modeled as a directed graph $G = (V, E)$ where V represents the set of WDM nodes and E is the set of edges. Each edge j contains l_j fibers which in turn contain a set of wavelength channels K.

We find in the spare capacity of the network a set of cycles P which are subject to two characteristics: the cycles are simple (i.e. the nodes of the cycle path are pairwise different) and restricted by a maximum physical length. The former characteristic provides manageable protection patterns, the latter keeps the delay of a connection during protection switching low and reduces also the vulnerability to multiple duct failures on a *p*-cycle.

Two incidence matrices are computed after finding the cycles. An entry $p_{i,j} \in \{0, 1\}$ of the first matrix indicates if the edge j is an element of *p*-cycle i. An entry $x_{i,j} \in \{0, 1\}$ of the second matrix indicates if a working connection on edge j is protectable by *p*-cycle i. Note that a bidirectional approach is also possible [1].

An edge j has a capacity of $c_j = l_j \times |K|$. On an edge j, w_j and s_j are the (given) number of working channels and the number of spare channels used by a *p*-cycle, respectively.

For the *p*-cycle configuration we are interested in the number of units (or copies) n_i of a cycle i that is needed (i.e. the *p*-cycle capacity).

The basic problem can be formulated as a feasibility problem without objective:

$$s_j = \sum_{i=1}^{|P|} p_{i,j}\, n_i, \quad \forall j \in E \tag{1}$$

$$w_j \le \sum_{i=1}^{|P|} x_{i,j}\, n_i, \quad \forall j \in E \tag{2}$$

$$w_j + s_j \le c_j, \quad \forall j \in E \tag{3}$$

$$n_i \in \{0, 1, 2, \ldots\}, \quad \forall i \in P \tag{4}$$

Constraints (1) determine the protection capacity allocation, constraints (2) ensure the working capacity to be protected, constraints (3) introduce the capacity restriction on the edges and by (4) we require integer *p*-cycle units.

We can add to the basic problem a desired objective, e.g. minimize the used protection resources [2]:

$$\min \sum_{j=1}^{|E|} s_j \tag{5}$$

For the tradeoff investigation we introduce a modification of the objective which is controled by a parameter α:

$$\min(1 - |\alpha|) \sum_{j=1}^{|E|} s_j + \alpha \frac{1}{M} f \tag{6}$$

where M is a large constant and f is a function with $f < M$, so that it cannot interfere with a protection capacity unit. Using $\alpha = 0$ we obtain objective (5). Note at this point that in using this formulation it is important to set the objective tolerance of the problem solver appropriately.

In addition to the basic model we introduce variables indicating that a *p*-cycle is taken:

$$t_i \in \{0, 1\}, \quad \forall i \in P \tag{7}$$

These constraints ensure that $t_i = 1$ for a selected cycle i (i.e. $n_i > 0$) and $t_i = 0$ otherwise:

$$n_i / N \le t_i \le n_i, \quad \forall i \in P \tag{8}$$

where $N \ge \max_{j \in E} c_j$ is a large constant.

We propose these functions for f:

1) The function $f = f_{sc} = \sum_{i=1}^{|P|} t_i$. It calculates the number of selected cycles.
2) The minimax function $f = f_{mm} = x_{max}$ with a new variable x_{max}, obeying $0 \le x_{max} < M$ and

$$\sum_{j=1}^{|E|} x_{i,j} t_i \le x_{max}, \quad \forall i \in P. \tag{9}$$

This function calculates the maximum working capacity coverage of the *p*-cycles, i.e. the maximum value of the sum of the $x_{i,j}$ values among all selected cycles i.

It has been shown that longer p-cycles (also having more working coverage) will be chosen in a capacity efficient optimization [2,7]. However, longer p-cycles and p-cycles covering more working capacity, also include more double failure cases.

We can influence the model by setting α to the desired optimization aim:

- There can be several solutions to the problem (1)-(4) and (5) with the same objective. These solutions, however, can yield different double failure performances. If attaining minimal protection capacity is the primary objective, we can try to find out a solution among the capacity-minimal which has most p-cycles, and thus, can be better at double failures (Sec. II).
  This can be done by setting $\alpha = \frac{1}{2}$ and $f = f_{sc}$.
- The value of f can be the primary aim, e.g., as an influence factor to the survivability of double failures or for the number of cycles. Capacity is not optimized, but the capacity constraints should be fulfiled. This can be done by setting $\alpha = 1$ and $\alpha = -1$ for minimizing f and maximizing f, respectively.

Compared with the basic problem ($\alpha = 0$) in the case study, these additions to the basic problem do not make the computation times very long.

IV. Case Study

As in [2], we optimize the pan-European COST 239 network with 11 nodes and 26 ducts. As the network has a high average nodal degree of 4.7, it can have high double failure survivability. The demand pattern is produced by the COST 239 network traffic matrix, where the entries are divided by $10^{-1} \times 2.5$ Gbit/s and interpreted then as lightpath demands (yielding 1760 bidirectional lightpath demands). Asymmetric entries in the matrix are set to the higher value.

The working connections are routed on the minimum hop path in the network. The bidirectional demands take the same path for both directions. The number of fibers is for every edge $l_j = 2, \forall j \in E$ and the number of wavelengths of each fiber is $|K| = 128$.

We develop an upper bound measure for the loss caused by a double fiber duct failure by counting:

i) all working links which are affected by the first failure and whose p-cycle is affected by the second failure
ii) all working links which are affected by the second failure and whose p-cycle is already used.

This way we calculate an upper bound loss for all the straddling/on-cycle failure possibilities except for the straddling 1/on-cycle 0 cases (Fig. 2 (c)-(f)). The reason for this is that although in these cases the straddling link that can survive taking the failure-free part of the p-cycle, it is counted as lost in ii).

Based on this loss calculation we define two metrics:

- $\overline{L}$: The average loss over all double failure events
- $\overline{R}$: The average restorability over all double failure events

The restorability of a double failure (i, j) is defined in this paper (similar to [4]) as the portion of all working links $w_i + w_j$ on the ducts i and j that are simultaneously affected by a double failure and survive this failure:

$$R_{(i,j)} = 1 - \frac{\text{Loss}(i,j)}{w_i + w_j} \qquad (10)$$

Fig. 5 depicts the mean number of lost unidirectional working capacity units $\overline{L}$ over the allowed maximum physical p-cycle length. If the highest number of p-cycles is chosen ($\alpha = -1$ and $f = f_{sc}$), the average loss during double failures is much less than for both the capacity-only optimization ($\alpha = 0$), and the minimal number of cycles optimization ($\alpha = 1$ and $f = f_{sc}$). Choosing a maximal number of cycles for a capacity-optimal solution ($\alpha = \frac{1}{2}$ and $f = f_{sc}$) yields nearly no improvement on the loss. The loss tends to be higher for longer maximum physical p-cycle lengths. This can be explained by the fact that longer p-cycles, also having more working capacity coverage, can become more vulnerable to double failures.

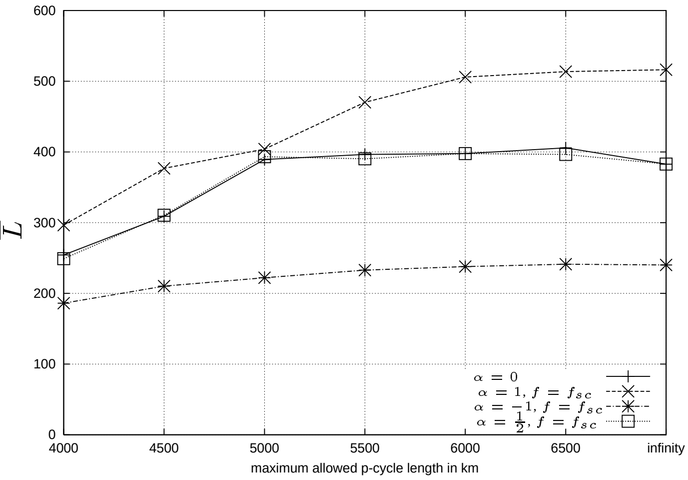

Fig. 5. The mean number of lost unidirectional paths $\overline{L}$ over the allowed maximum physical p-cycle length.

The mean restorability of approximately 70% (Fig. 6) for the maximum number of cycles ($\alpha = -1$ and $f = f_{sc}$) is reasonably better than for the capacity-optimal design ($\alpha = 0$) and the minimum number of cycles design ($\alpha = 1$ and $f = f_{sc}$). Keeping the maximum working capacity coverage low ($\alpha = 1$ and $f = f_{mm}$) can even achieve better mean restorability values (approximately 76%).

In Fig. 7 we clearly see the price for obtaining a better restorability. An optimization using as many cycles as possible ($\alpha = -1$ and $f = f_{sc}$) chooses one to two orders of magnitude more cycles than in the other cases. It is remarkable that the solver in the capacity-only optimization ($\alpha = 0$) has chosen few cycles, rather near to the minimum possible number of cycles (which is only 3 unidirectional cycles). Around 30 cycles are used when minimizing the maximum working capacity coverage ($\alpha = 1$ and $f = f_{mm}$).

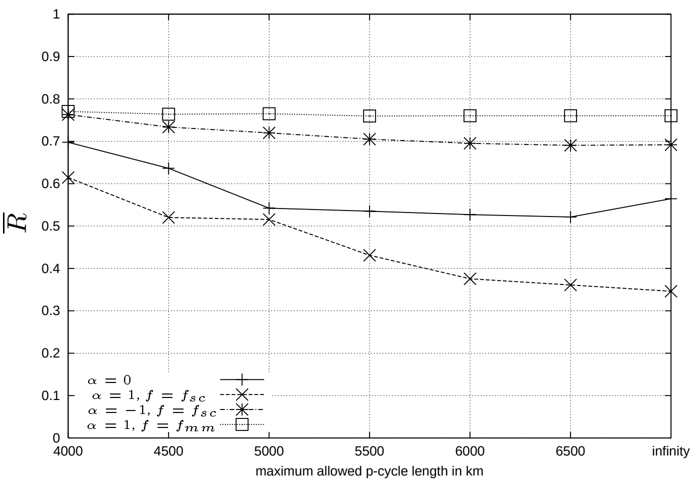

Fig. 6. The mean restorability $\overline{R}$ and the minimum restorability R_{min} of a double failure event.

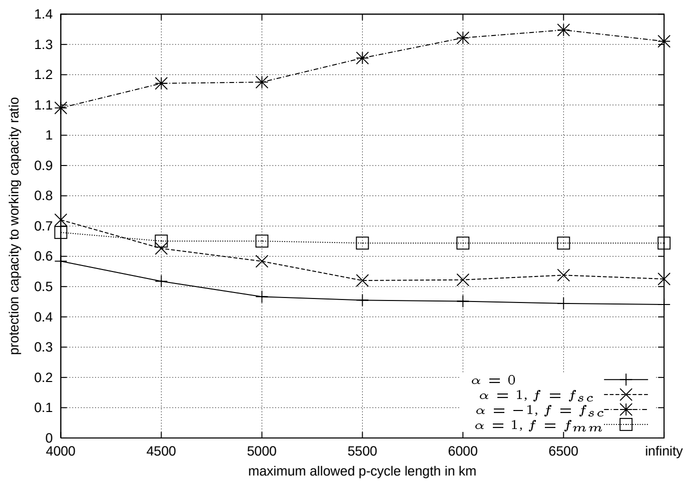

Fig. 8. The efficiency ratio over the allowed maximum physical *p*-cycle length.

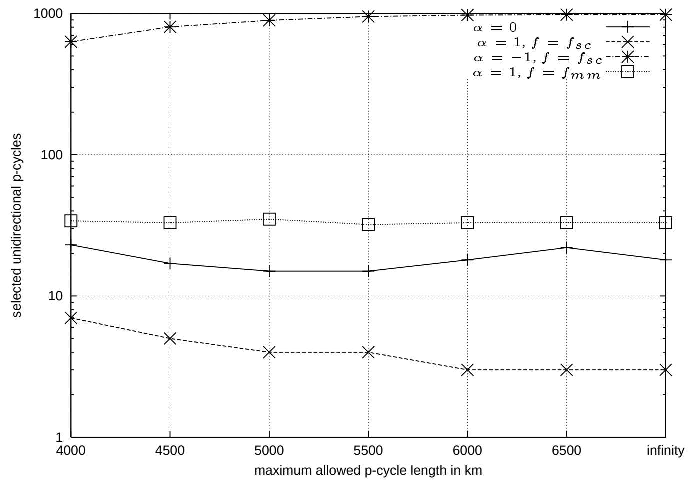

Fig. 7. The number of selected *p*-cycles over the allowed maximum physical *p*-cycle length.

Fig. 8 shows the efficiency ratio $\sum_j s_j / \sum_j w_j$ over the allowed maximum physical *p*-cycle length. The capacity-only optimization ($\alpha = 0$) and minimum possible number of cycles optimization ($\alpha = 1$ and $f = f_{sc}$) are very capacity efficient (below 75%). The minimax working capacity coverage optimization ($\alpha = 1$ and $f = f_{mm}$) comes near to these efficiency values. Using a maximal number of cycles ($\alpha = -1$ and $f = f_{sc}$) consumes more than an additional 100% working capacity for protection.

V. Conclusions

In this paper we analyzed the loss that dual failures cause to *p*-cycles. We developed an upper bound measure for the loss and the restorability upon double failures. We proposed several designs where the network is optimized concerning capacity, and/or has a minimum or maximum number of cycles. Using these designs for a case study of a pan-European network, we conclude the following behavior:

- We calculated higher mean dual failure loss if longer *p*-cycles are allowed for the optimal selection.
- When maximizing the number of cycles we obtain high dual failure restorability (approx. 70%). However, this can require a huge amount of cycles and high protection capacity.
- When minimizing the number of cycles, we obtain few cycles (less than ten) which keeps the administrative effort simple.
- The capacity optimal design and the minimal number of cycles design can provide a dual failure restorability of 50%.
- Minimizing the maximum working capacity coverage of selected *p*-cycles is promising to achieve even better mean dual failure restorability, while still providing capacity efficiency.

Further work explores the dependence of the dual failure restorability on the topology.

The author would like to acknowledge the suggestions of the anonymous reviewers and of C. Gruber and M. Scheffel.

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